- I can prove and apply properties of perpendicular bisectors of a triangle.
- I can prove and apply properties of angle bisectors of a triangle.

Vocabulary		
concurrent	point of concurrency	circumcenter of a triangle
circumscribed	incenter of a triangle	inscribed

Common Core
- **CC.9-12.G.C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
- **CC.9-12.G.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straight edge, string, reflective devices, paper folding, dynamic geometry software, etc.)
- **CC.9-12.G.MG.3** Apply geometric methods to solve design problems.

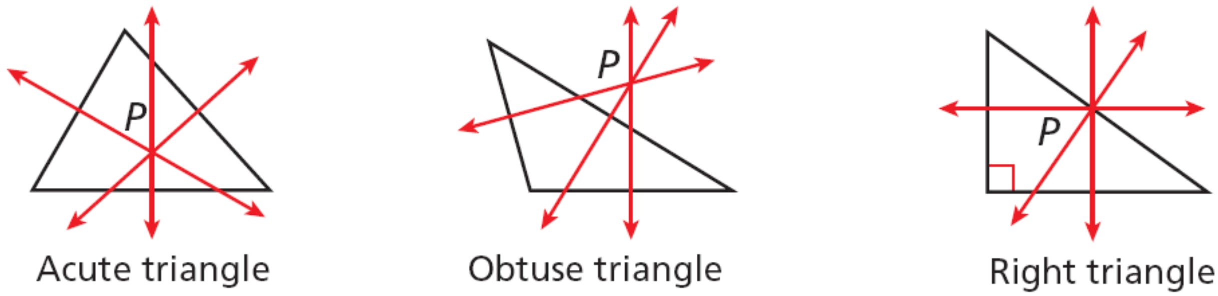
Sketchpad: Perpendicular Bisectors in a Triangle. (Upload sketch to google.classroom)

Since a triangle has three sides, it has three perpendicular bisectors. When you construct the perpendicular bisectors, you find that they have an interesting property.

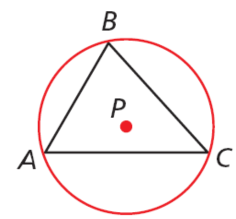
Helpful Hint

The perpendicular bisector of a side of a triangle does not always pass through the opposite vertex.

The circumcenter can be inside the triangle, outside the triangle, or on the triangle.



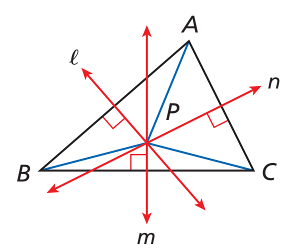
The circumcenter of ΔABC is the center of its circumscribed circle. A circle that contains all the vertices of a polygon is **circumscribed** about the polygon.



PROOF

Circumcenter Theorem

Given: Lines ℓ, m, and n are the perpendicular bisectors of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively.
Prove: $PA = PB = PC$

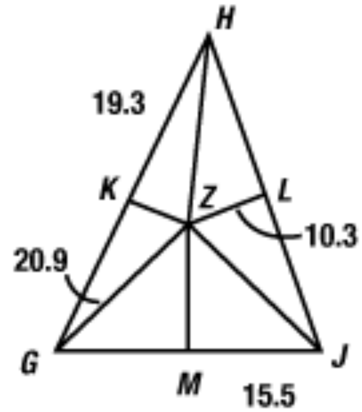


Proof:

P is the circumcenter of $\triangle ABC$. Since P lies on the perpendicular bisector of $\overline{AB}$, $PA = PB$ by the Perpendicular Bisector Theorem. Similarly, P also lies on the perpendicular bisector of $\overline{BC}$, so $PB = PC$. Therefore $PA = PB = PC$ by the Transitive Property of Equality.

<u>Video Example 1</u>. $\overline{KZ}, \overline{LZ}, \& \overline{MZ}$ are perpendicular bisectors of $\triangle GHJ$. Find JZ & GM.

Find *JZ* and *GM*.



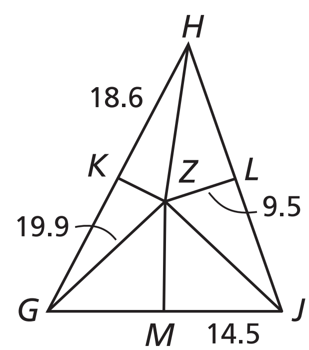
1 Using Properties of Perpendicular Bisectors

$\overline{KZ}$, $\overline{LZ}$, and $\overline{MZ}$ are the perpendicular bisectors of $\triangle GHJ$. Find HZ.

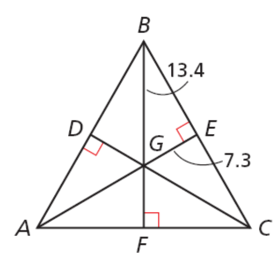
Z is the circumcenter of $\triangle GHJ$. By the Circumcenter Theorem, Z is equidistant from the vertices of $\triangle GHJ$.

$HZ = GZ$ *Circumcenter Thm.*

$HZ = 19.9$ *Substitute 19.9 for GZ.*

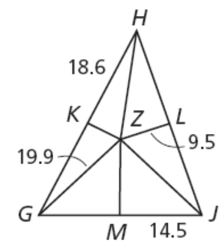


<u>Example 1</u>. $\overline{DG}, \overline{EG}, \& \overline{FG}$ are the perpendicular bisectors of $\triangle ABC$. Find GC.

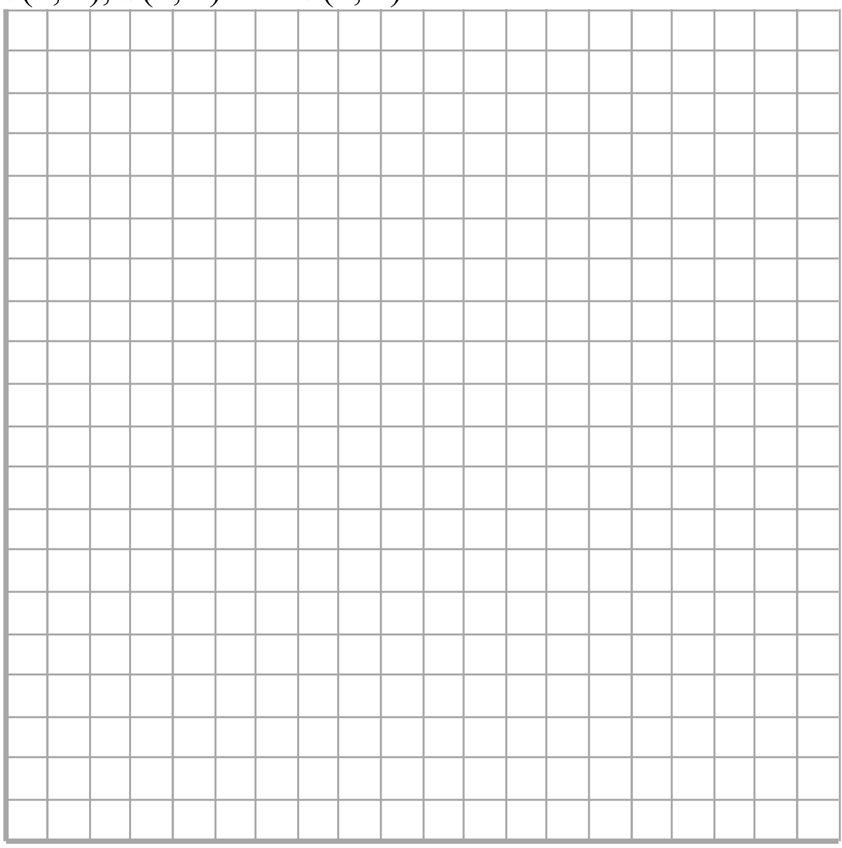


Guided Practice. Find the following.

1. GM **2.** GK **3.** JZ



Video Example 2. Find the circumcenter of $\triangle JSO$ with vertices J(8, 0), S(0, 6) and O(0, 0).



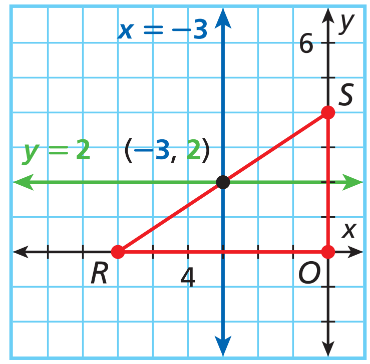
2 Finding the Circumcenter of a Triangle

Find the circumcenter of $\triangle RSO$ with vertices $R(-6, 0)$, $S(0, 4)$, and $O(0, 0)$.

Step 1 Graph the triangle.

Step 2 Find equations for two perpendicular bisectors.

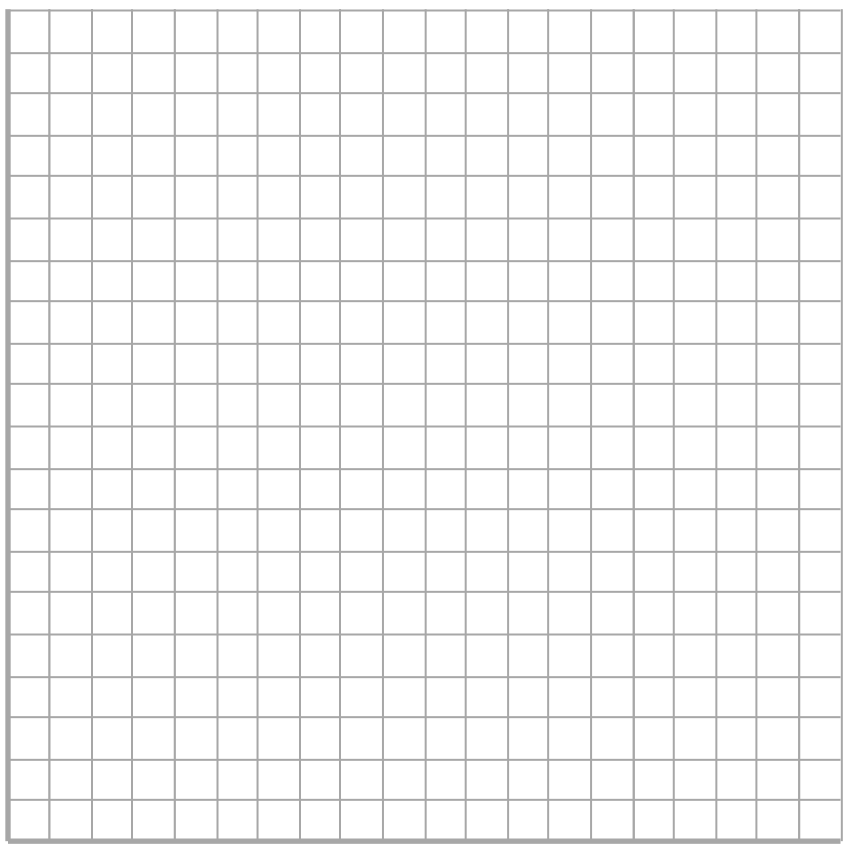
Since two sides of the triangle lie along the axes, use the graph to find the perpendicular bisectors of these two sides. The perpendicular bisector of $\overline{RO}$ is $x = -3$, and the perpendicular bisector of $\overline{OS}$ is $y = 2$.



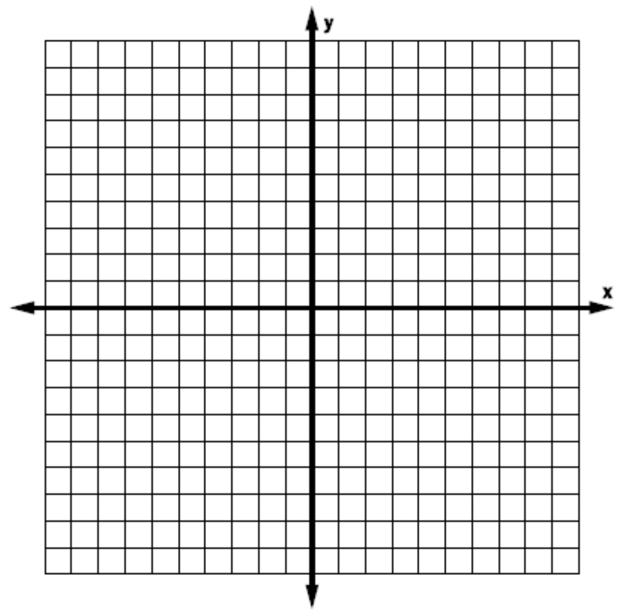
Step 3 Find the intersection of the two equations.

The lines $x = -3$ and $y = 2$ intersect at $(-3, 2)$, the circumcenter of $\triangle RSO$.

<u>Example 2</u>. Find the circumcenter of ΔHJK with vertices $H(0, 0)$, $J(10, 0)$, and $K(0, 6)$.



4. Find the circumcenter of ΔGOH with vertices $G(0, -9)$, $O(0, 0)$, and $H(8, 0)$.



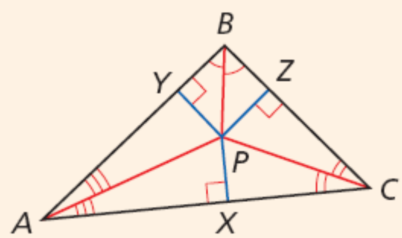
Sketchpad: Angle Bisectors in a triangle.

A triangle has three angles, so it has three angle bisectors. The angle bisectors of a triangle are also concurrent. This point of concurrency is the **<u>incenter of the triangle</u>** .

Theorem 5-2-2 **Incenter Theorem**

The incenter of a triangle is equidistant from the sides of the triangle.

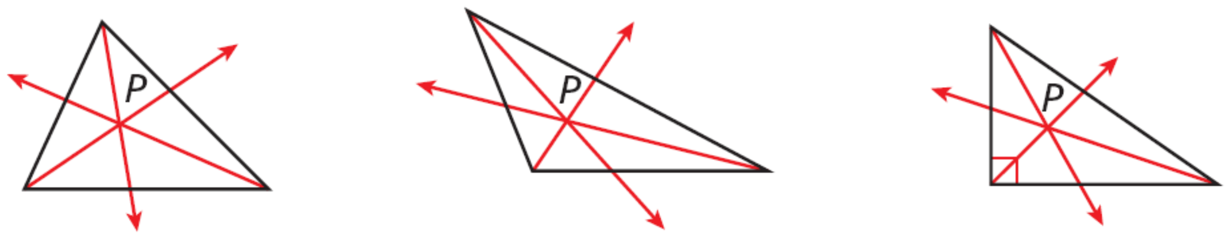
$PX = PY = PZ$



Remember!

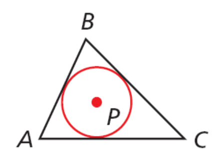
The distance between a point and a line is the length of the perpendicular segment from the point to the line.

Unlike the circumcenter, the incenter is always inside the triangle.



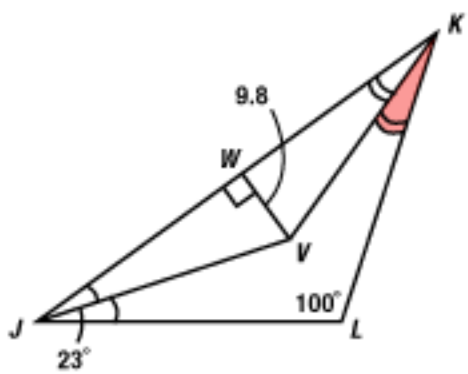
Acute triangle   Obtuse triangle   Right triangle

The incenter is the center of the triangle's *inscribed circle*. A circle **inscribed** in a polygon intersects each line that contains a side of the polygon at exactly one point.

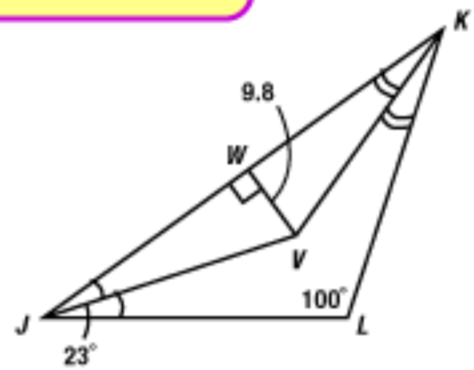


Vidoe Example 3. $\overline{JV}$ & $\overline{KV}$ are angle bisectors of $\triangle JKL$. Find each measure.

m∠VKL

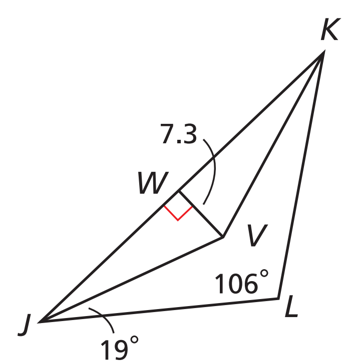


the distance from V to $\overline{KL}$



3 Using Properties of Angle Bisectors

$\overline{JV}$ and $\overline{KV}$ are angle bisectors of $\triangle JKL$. Find each measure.



A **the distance from V to $\overline{KL}$**

V is the incenter of $\triangle JKL$. By the Incenter Theorem, V is equidistant from the sides of $\triangle JKL$.

The distance from V to $\overline{JK}$ is 7.3.
So the distance from V to $\overline{KL}$ is also 7.3.

B **$m\angle VKL$**

$m\angle KJL = 2m\angle VJL$ — *$\overline{JV}$ is the bisector of $\angle KJL$.*

$m\angle KJL = 2(19°) = 38°$ — *Substitute 19° for $m\angle VJL$.*

$m\angle KJL + m\angle JLK + m\angle JKL = 180°$ — *$\triangle$ Sum Thm.*

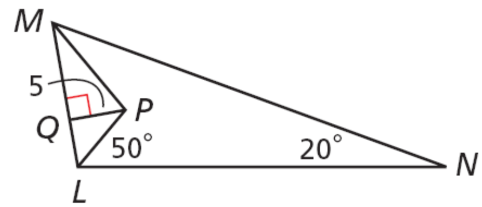
$38 + 106 + m\angle JKL = 180$ — *Substitute the given values.*

$m\angle JKL = 36°$ — *Subtract 144° from both sides.*

$m\angle VKL = \frac{1}{2}m\angle JKL$ — *$\overline{KV}$ is the bisector of $\angle JKL$.*

$m\angle VKL = \frac{1}{2}(36°) = 18°$ — *Substitute 36° for $m\angle JKL$.*

Example 3. $\overline{MP}$ & $\overline{LP}$ are angle bisectors of ΔLMN. Find the distance from P to $\overline{MN}$ & $\measuredangle PMN$.

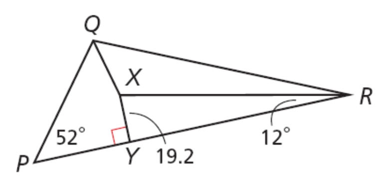


5-2 Bisectors in a triangle (pp 323) 12, 15-19.

<u>Guided Practice</u>**.** $\overline{QX}$ **and** $\overline{RX}$ **are angle bisectors of** ΔPQR**. Find each measures**

4. The distance from X to $\overline{PQ}$.

5. $m\measuredangle PQX$.



Refer to video example 4 (my.hrw.com)

Community Application

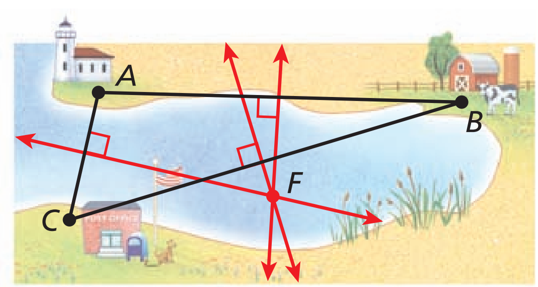
The building contractor for the city wants to build a public restroom in the park between three recreation areas. Draw a sketch to show where the restroom should be positioned so that it is the same distance from all three areas. Justify your sketch.

4 Community Application

For the next Fourth of July, the towns of Ashton, Bradford, and Clearview will launch a fireworks display from a boat in the lake. Draw a sketch to show where the boat should be positioned so that it is the same distance from all three towns. Justify your sketch.

Let the three towns be vertices of a triangle. By the Circumcenter Theorem, the circumcenter of the triangle is equidistant from the vertices.

Trace the outline of the lake. Draw the triangle formed by the towns. To find the circumcenter, find the perpendicular bisectors of each side. The position of the boat is the circumcenter, *F*.

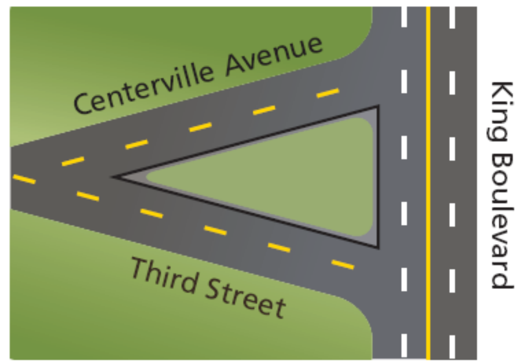


Teacher: How many sides does a circle have?
Student: Two—inside and outside!

Things are not as simple as they seems at first. - *Edward Thorp*

<u>Example 4</u>. A city planner wants to build a new library between a school, a post office, and a hospital. Draw a sketch to show where the library should be placed so it is the same distance from all three buildings.

6. <u>Guided Practice</u>. A city plans to build a firefighters' monument in the park between three streets. Draw a sketch to show where the city should place the monument so that it is the same distance from all three streets. Justify your sketch.



5-2 Bisectors in a triangle (pp 323) 12, 15-22, 32, 37, 38.